NORTHWESTERN UNIVERSITY TRANSPORTATION CENTER

Northwestern | Economics

Anticipatory Pricing to Manage "Flow Breakdown"

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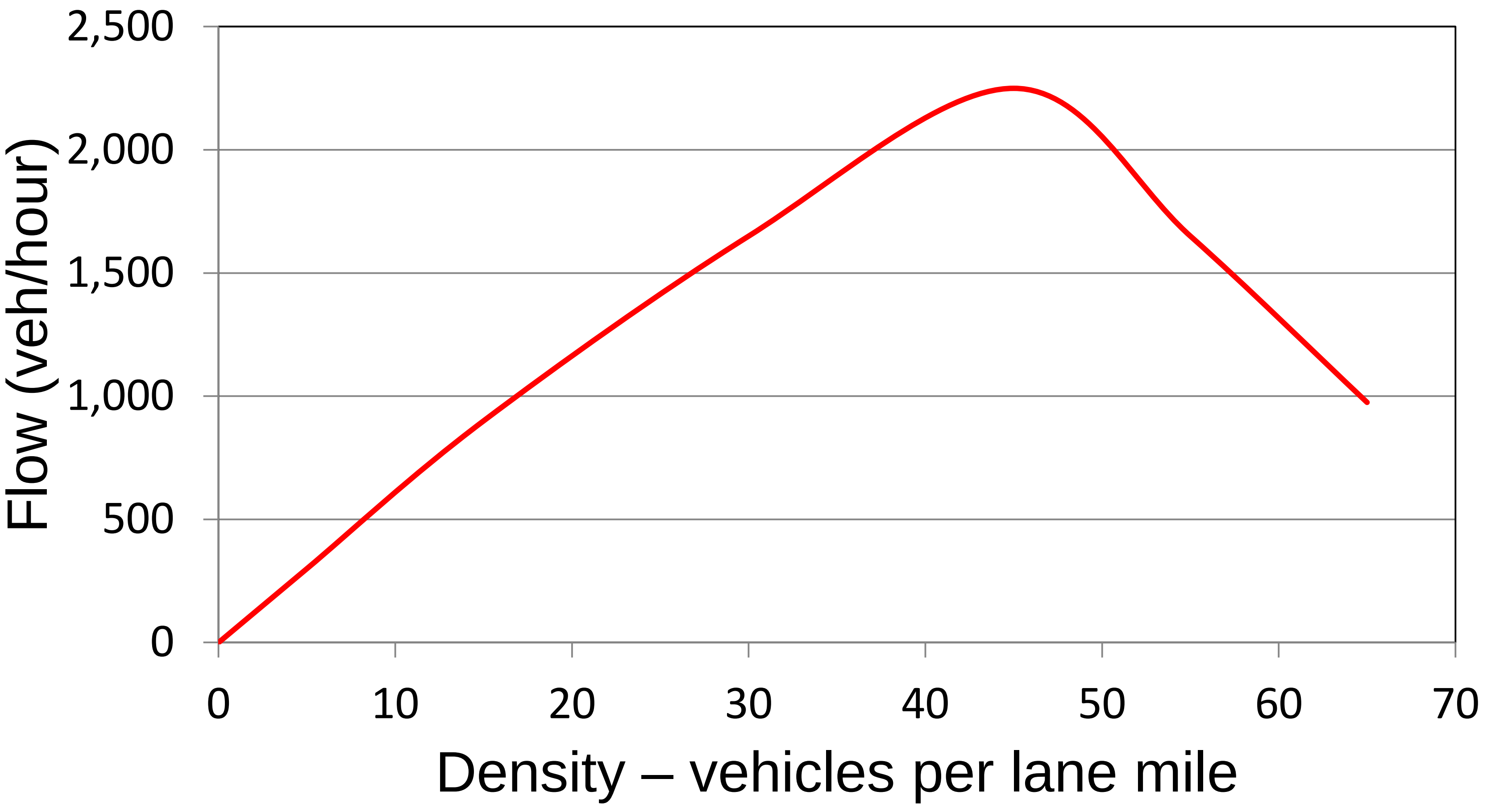
- Flow = density x speed

Fundamental diagram of traffic



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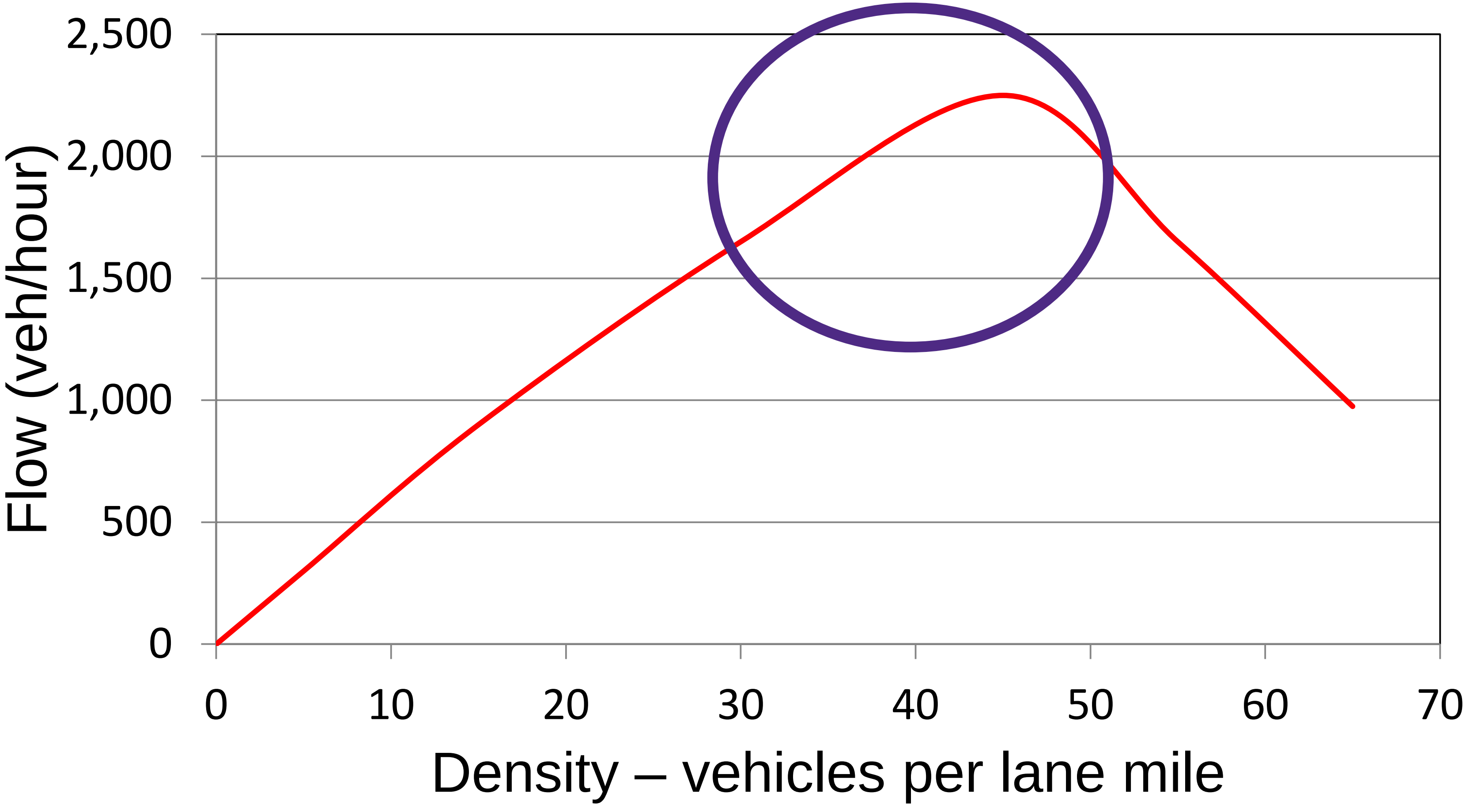
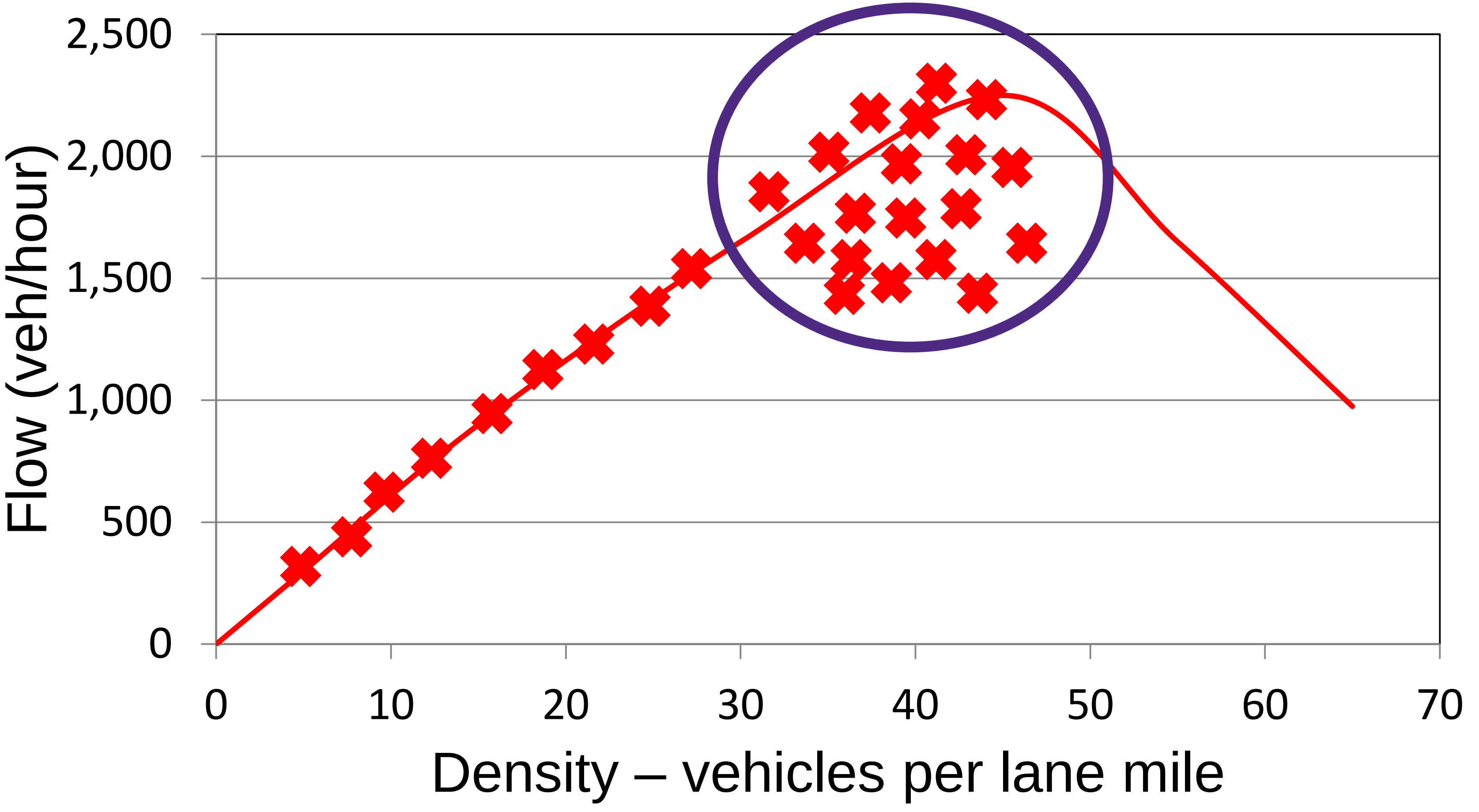


“Flow Breakdown”



Causes

- Weaving between lanes
- Excessively slow vehicles
- Aggressive driving
- Sharp brakeing
- Unusual weather
- Unusual visual distraction

Features

- Does not occur everyday (probabilistic)
- Precursor action more likely to result in breakdown at higher densities/flows
- Occurs when highway is operating at less than theoretical capacity

What it is not

- Backup caused by a downstream bottleneck now affecting this link

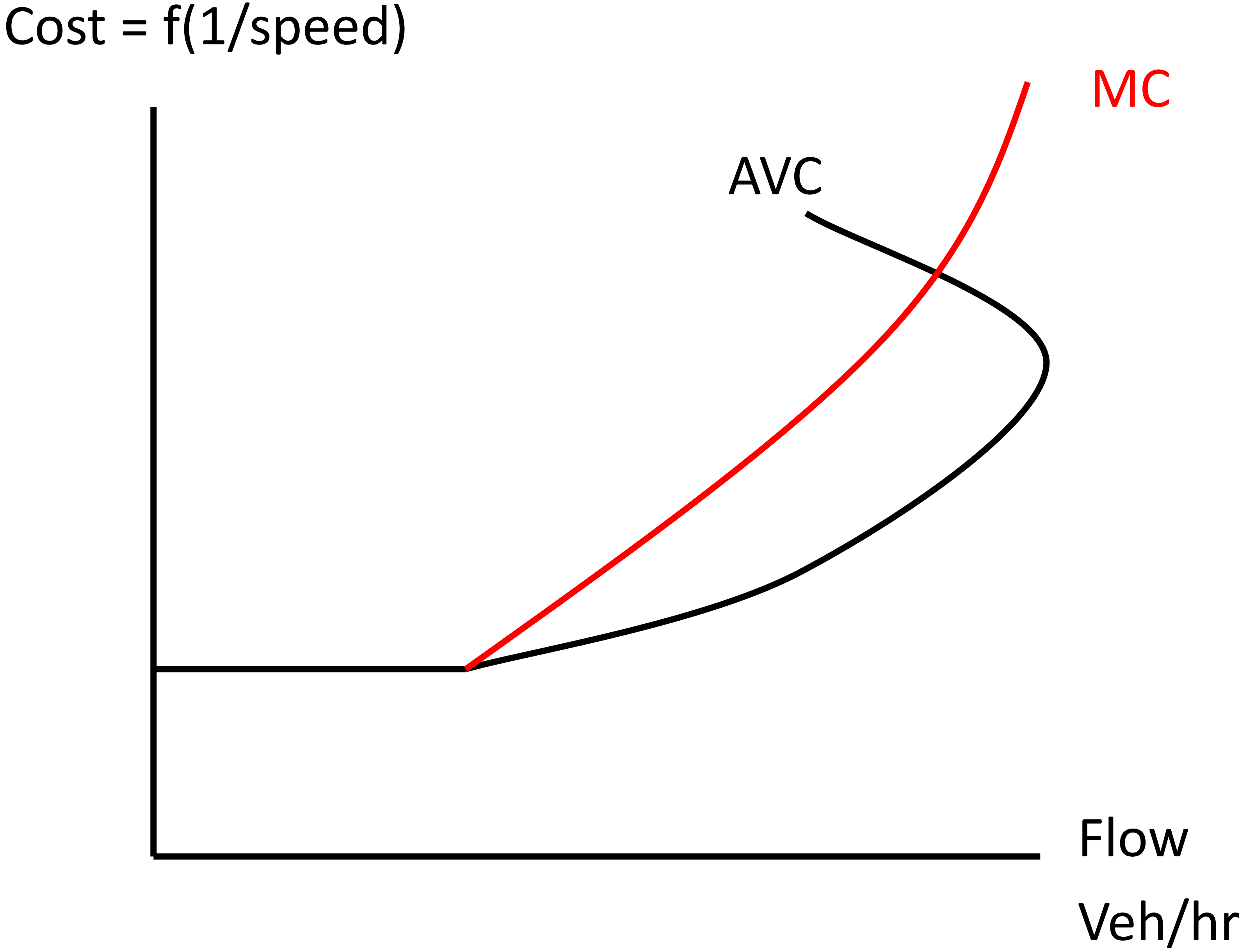
- Random traffic crashes that close some or all lanes

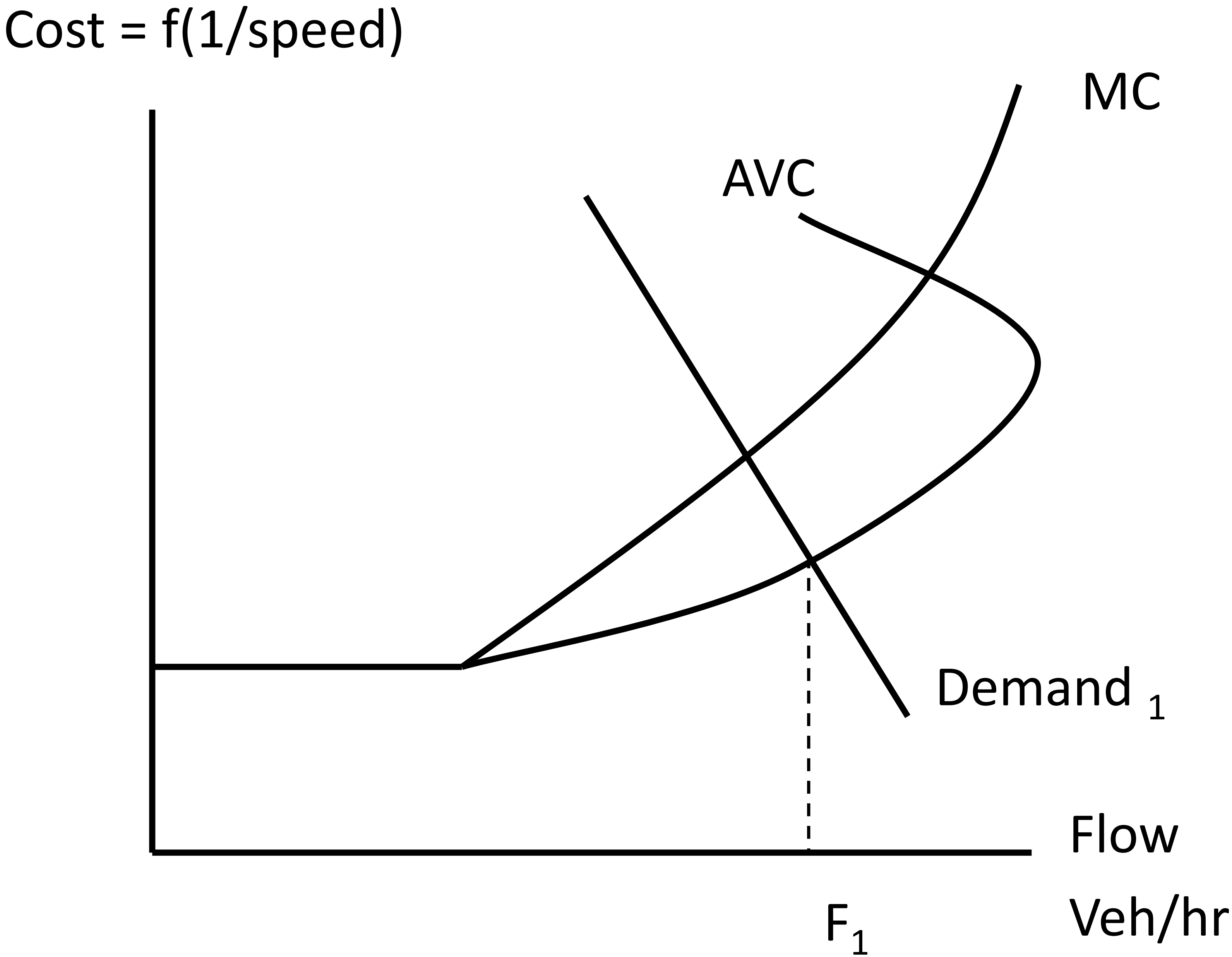
- Oversaturation of the link

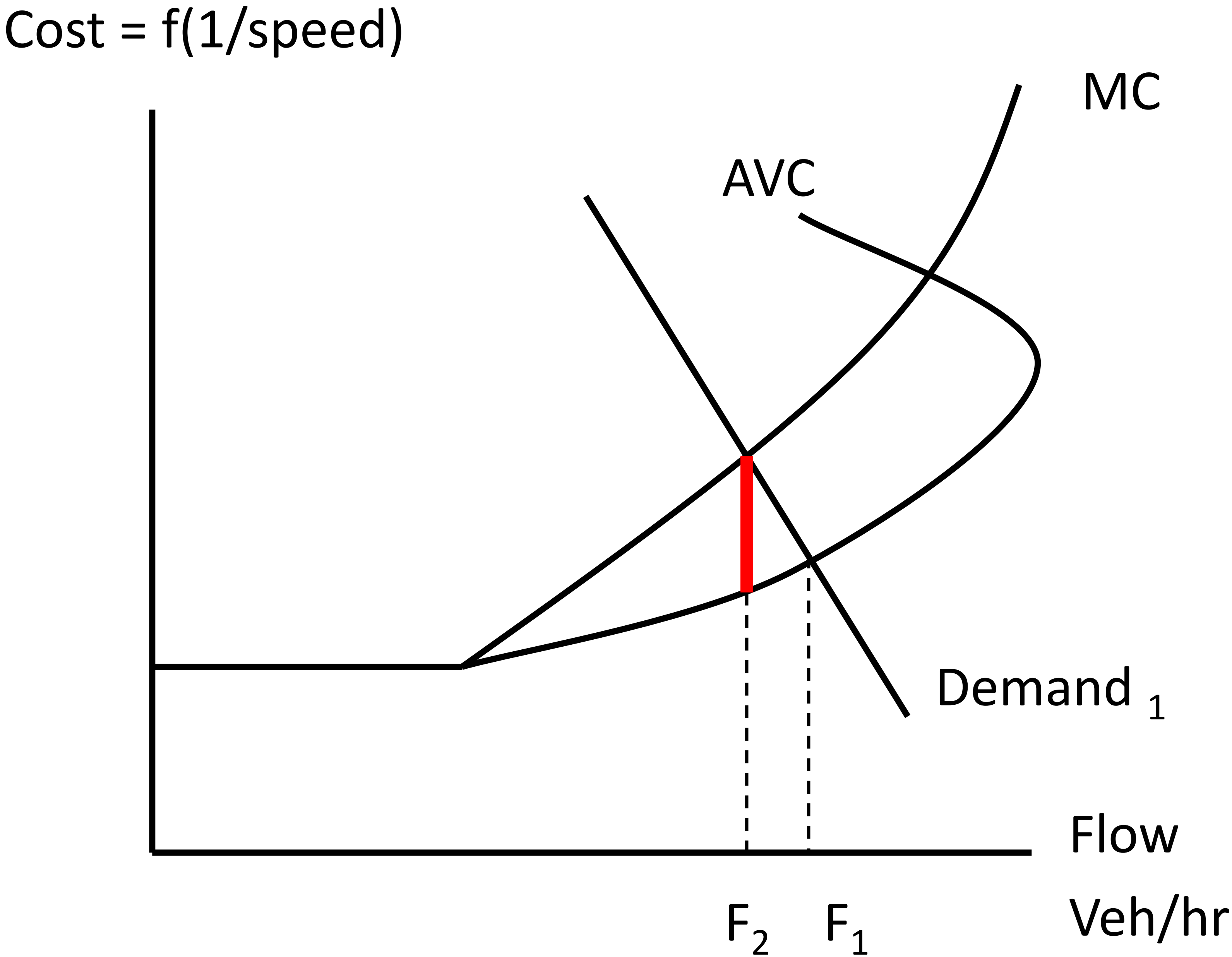
Congestion pricing I

- Flow less than maximum capacity:
  - “normal congestion” (economists)
  - “undersaturated” (engineer)

- Has stable density/speed/flow relationship


Cost = f(1/speed)
MC
AVC
Flow
Veh/hr


Cost = f(1/speed)
MC
AVC
Demand 1
Flow
Veh/hr
F1



Cost = f(1/speed)
MC
AVC
Demand 1
Flow
Veh/hr
F2
F1

Congestion pricing II

- (In)flow greater than maximum capacity:
  - “hypercongested” (economists)
  - “oversaturated” (engineer)

- “Bottleneck” model
  - Dates to Vickrey in the 1960s
  - Modern version started with Arnott, De Palma and Lindsey, 1990

Congestion pricing II

- Bottleneck of fixed capacity
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- Drivers endogenously select their departure time from “home” (discussed in a minute)
- May arrive at “work” earlier or later than they would like (disutility from this variation)
- Introduction of pricing shortens smooths inflow and makes drivers better off

Our objective

- Adapt the bottleneck model to deal with situations where the equilibrium flow is less than theoretical capacity:
  - “Good days” when drivers encounter no congestion
  - “Bad days” when breakdown occurs (a bottleneck become binding) and drivers encounter congestion in the form of a queue
- Make the probability of a “bad day” endogenous

How are you going to price?

- Option 1 – real time dynamic ex-post pricing to help highway recover on "bad days"
  - Need alternative routes
  - And/or people delay or not make trips or change mode

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  - And/or people delay or not make trips or change mode
- Option 1A – upstream sensors and traffic prediction models guess if and where breakdown is likely and price accordingly – Dong and Mahmassani (2013)

How are you going to price?

- Option 2 – Anticipatory pricing:
  - Same price on both good and bad days
  - Price set in advance so drivers know it in making departure time decisions
  - Drivers know in advance how traffic performs on both good and bad days
  - Drivers know the – endogenous – probability of a bad day
  - Hence choose their departure time from home

THE MODEL

Simplifications

- Morning peak
- Fixed "totally inelastic" number of commuters (Q) in single-occupancy cars
- Homogenous drivers (same utility function and tastes)
- Same desired arrival time at work (t*)

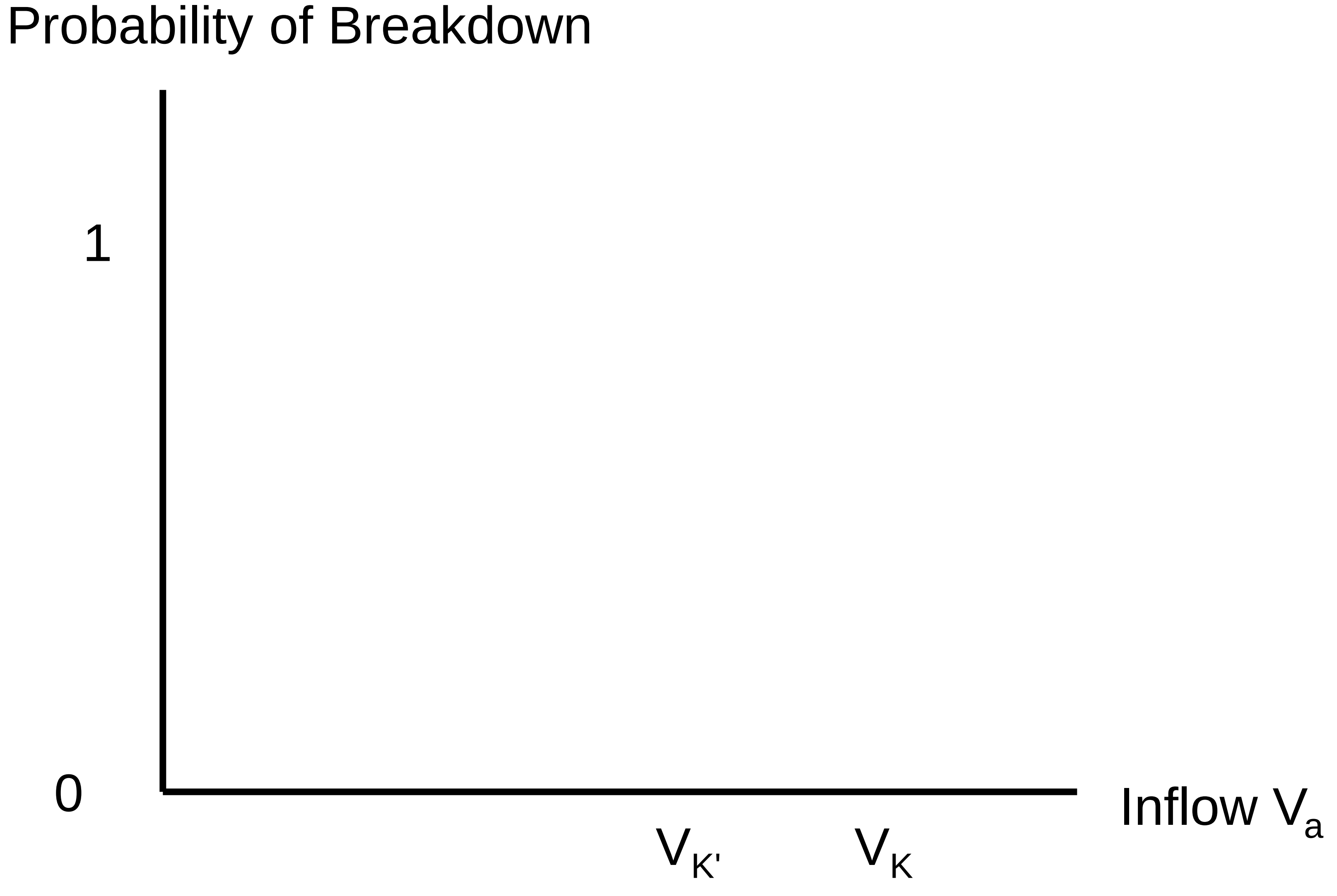
Simplifications

- Single link between “home” and “work”
- “Home” is located immediately before a possible bottleneck
- “Work” is located immediately after the bottleneck
- So, free-flow travel time and vehicle operating costs are normalized to zero

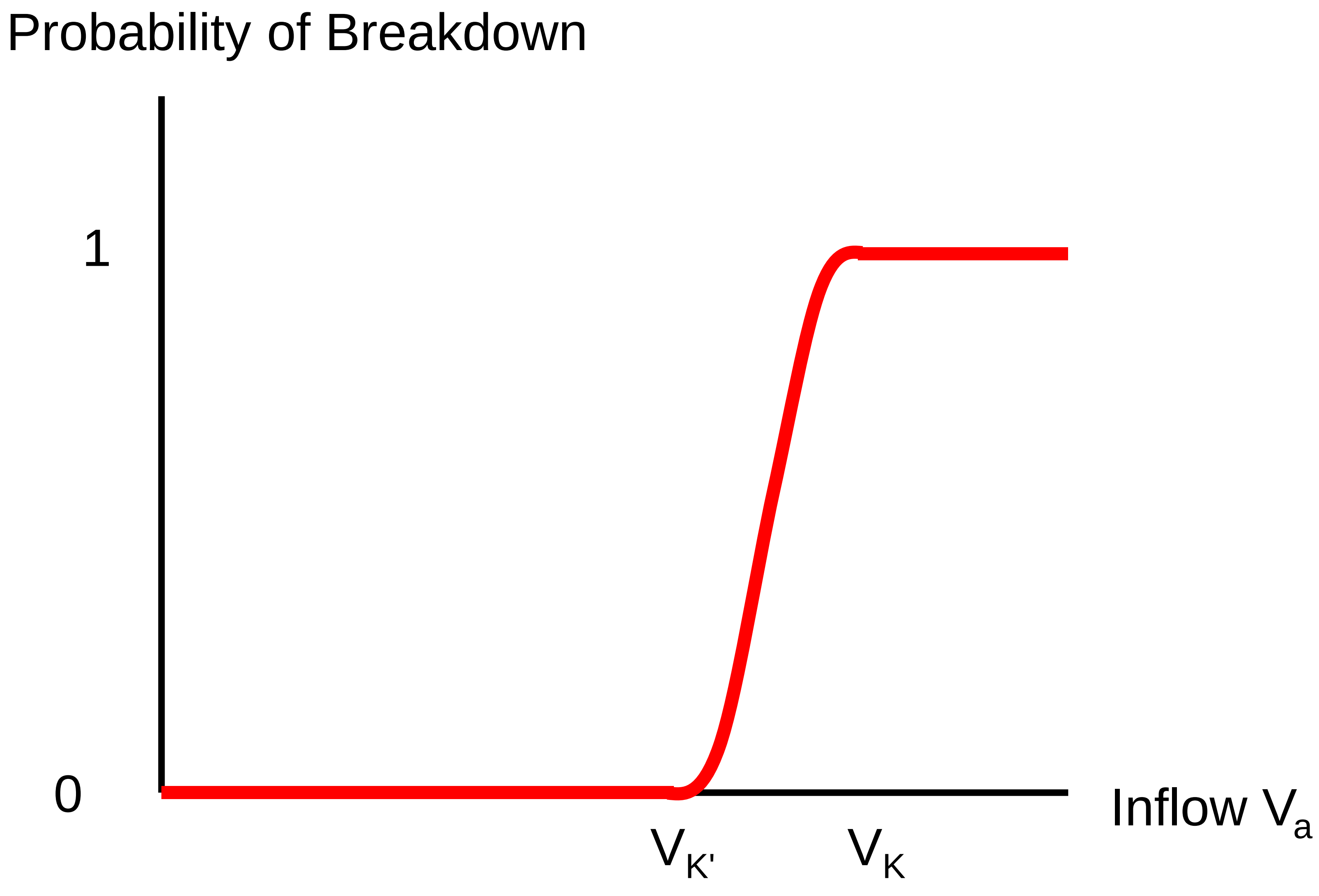
Highway technology

- (Out)flow capacity of the bottleneck in non-breakdown state (V_K) is not binding on inflow $V_a(t)$ for any value of t on a good day
- If breakdown occurs, capacity falls to $V_{K'}$ which is binding on $V_a(t)$ for at least some values of t
- Then a vertical queue develops
- Highway remains in breakdown state until queue totally dissipates, then it resets

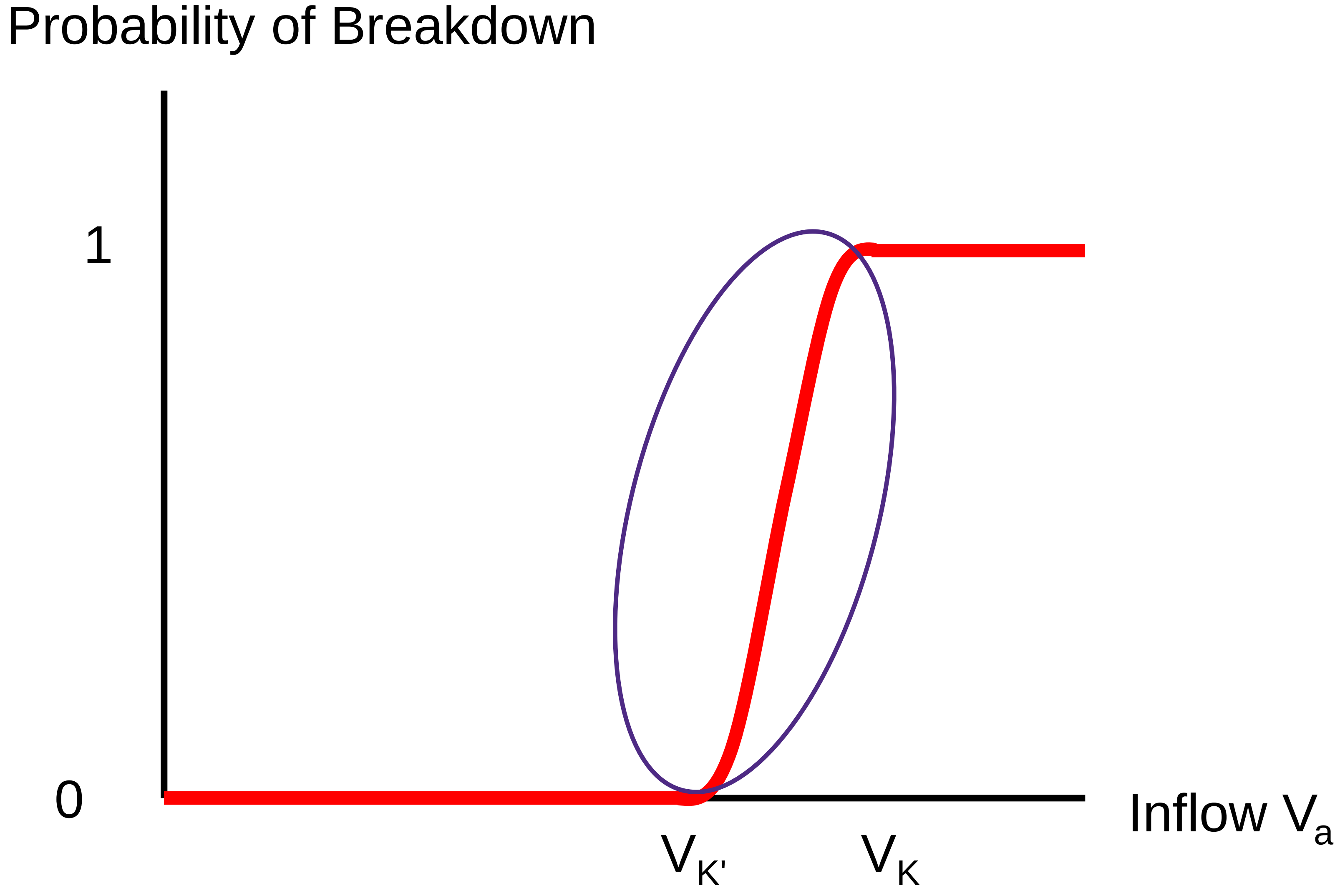
Probability of breakdown



Probability of breakdown



Probability of breakdown



When does breakdown occur?

- We will show that inflow $V_a(t)$ is highest earlier in the peak period
- Breakdown probability based on this maximum inflow
- If breakdown occurs at all, it happens immediately at the start of the peak
- Random drawing each morning based on endogenous probability
- Because on a bad day the queue does not dissipate until the end of peak, cannot have highway "recover" and then face possibility of relapse into breakdown
- So breakdown either at beginning of peak or not at all

Driver decision making

- All desire to get to work at t*
- Chose departure time from home in continuous time t
- Departure time can be earlier than, same as, or later than t*
- Objective is to minimize disutility (generalized cost) of their trip
- Equilibrium conditions:
  - No driver can shift departure time to improve their welfare
  - (implies that all drivers face same generalized cost)
  - Everyone who leaves home gets to work!

Generalized cost

- Some things normalized to zero
  - Free-flow travel time
  - Vehicle operating costs

1. Travel time delay (time in queue) valued at α
   - Note that on “good day” travel delay is zero
2. Schedule delay - work arrival time relative to t*
   - if arrive at work early valued at β
   - if arrive at work late valued at γ
   - usual assumption that $\beta < \alpha < \gamma$
3. Time-varying toll $\tau(t)$

NO-TOLL BASE CASE

Three groups of commuters

- Early: arrive at work early or exactly “on time” on both good and bad days

Early commuters

$$c_g(t) = p(V_a^{early})\ \{\alpha T_{DB}(t) + \beta\ [t^* - (t + T_{DB}(t))]\}$$
$$+ [1 - p(V_a^{early})]\ \beta\ (t^* - t)$$

Early commuters

$c_g(t) = p(V_a^{early})\ \{\alpha T_{DB}(t) + \beta\ [t^* - (t + T_{DB}(t))]\}$

$\qquad + [1- p(V_a^{early})]\ \beta\ (t^* - t)$

Solve by:

$$\frac{\partial c_g(t)}{\partial t} = 0$$

$$\frac{\partial T_{DB}(t)}{\partial t} = \frac{V_a^{early}}{V_{K'}} - 1$$

Early commuters

$$V_a^{early} = \left[\frac{\beta}{p\left(V_a^{early}\right)(\alpha - \beta)} + 1\right] V_{K},$$

Three groups of commuters

- Early: arrive at work early or exactly “on time” on both good and bad days

- Middle: arrive at work early or exactly on time on good days and late on bad days

Middle commuters

$$c_g(t) = p(V_a^{early})\,\{\alpha T_{DB}(t) + \gamma\,[t + T_{DB}(t) - t^*]\}$$
$$+\,[1- p(V_a^{early})]\,\beta\,(t^* - t)$$

Solve in similar fashion

Middle commuters

$$V_a^{middle} = \left[\frac{\left(1 - p\left(V_a^{early}\right)\right)\beta - \ p\left(V_a^{early}\right)\gamma}{p\left(V_a^{early}\right)(\alpha + \gamma\ )} + 1 \right] V_{K'}$$

- $V_a^{middle} < V_a^{early}$

Three groups of commuters

- Early: arrive at work early or exactly “on time” on both good and bad days
- Middle: arrive at work early or exactly on time on good days and late on bad days
- Late: arrive at work late on both good and bad days

Late commuters

$$c_g(t) = p(V_a^{early})\ \{\alpha T_{DB}(t) + \gamma\ [t + T_{DB}(t) - t^*)]\}$$
$$+\ [1- p(V_a^{early})]\ \gamma\ (t - t^*)$$

Solve in similar fashion

Late commuters

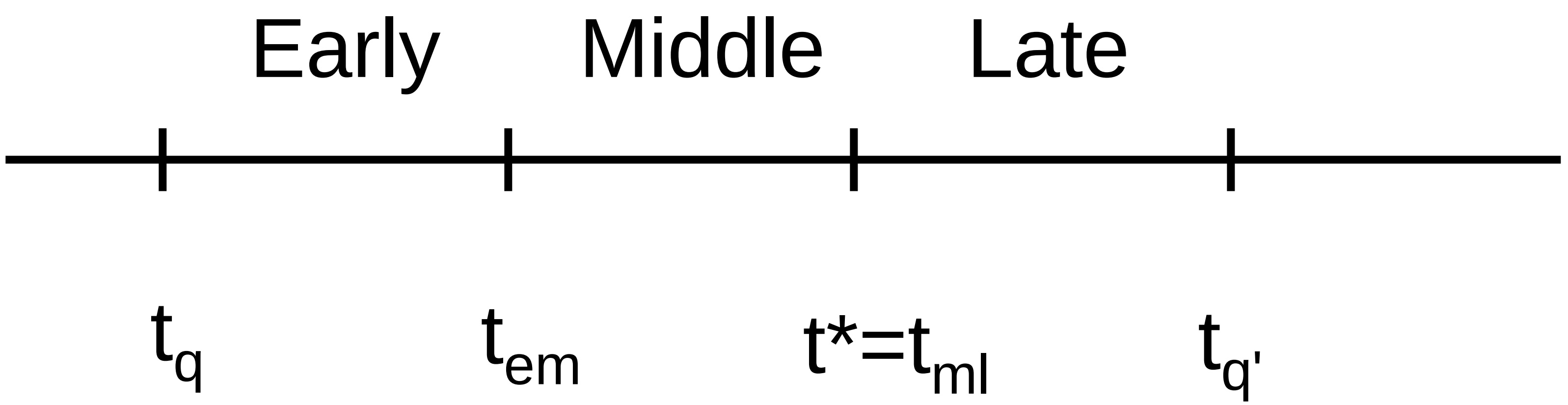
$$V_a^{late} = \left[\frac{-\gamma}{p\left(V_a^{early}\right)(\alpha+\gamma)} + 1\right] V_{K'}$$

- $V_a^{late} < V_a^{middle} < V_a^{early}$
- $V_a^{late} < V_{K'}$

The model

- Predetermined parameters: Q, α, β, γ, V_K, $V_{K'}$, distribution of $p(V_a^{early})$
- Just determined: V_a^{early}, V_a^{middle}, V_a^{late}
- Still to be determined:
  - t_q = departure time of earliest commuter
  - t_{em} = break point between early and middle group
  - (t_{ml} = t* = break point between middle and late group)
  - $t_{q'}$ = departure time of the last commuter
  - Q^{early}, Q^{middle}, Q^{late}

Time line (not to scale)



Can define a system of linear equations to solve for these

$$t_q = t^* - \frac{Q}{\left[1 + \frac{V_a^{middle}}{V_a^{late} - V_{K'}}\left(\frac{V_{K'}}{V_a^{early}} - 1\right)\right] V_{K'}}$$

$$t_{em} = \frac{V_{K'}}{V_a^{early}} t^* + \left(1 - \frac{V_{K'}}{V_a^{early}}\right) t_q$$

Can define a system of linear equations to solve for these

$$T_{DB}(t^*) = \frac{V_a^{middle}}{V_{K'}} t^* - \frac{V_a^{middle}}{V_{K'}} t_{em}$$

$$t_{q'} = t^* - \frac{V_{K'}}{V_a^{late} - V_{K'}} T_{DB}(t^*)$$

Q^{early}, Q^{middle}, Q^{late} follow from these

OPTIMAL FINE TOLLS

The standard bottleneck model

- When inflow > capacity even on a good day
- Set price schedule so there is a constant inflow equivalent to the bottleneck capacity (no queuing)
- For each driver the combined:
  - travel delay (eliminated by pricing)
  - schedule delay early or late
  - toll paid

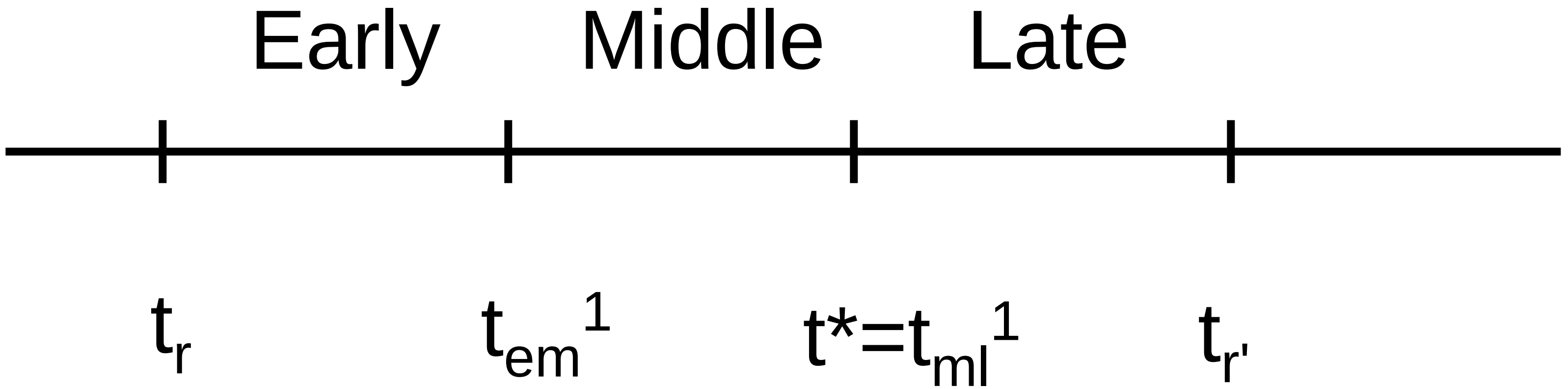
  is the same (i.e, the less the schedule delay, the higher the toll)

In our model

- Set price schedule to regulate inflow so it a constant rate equivalent to the maximum expected bottleneck capacity

$$V_a^1 = \max\{[1 - p(V_a)]V_a + p(V_a)V_{K'}\}$$

New time line (not to scale)



Solving the model

- Both the very first (at t_r) and very last driver (at $t_{r'}$) pay zero toll, but suffer:
  - Schedule delay early and zero traffic delay (on both good and bad days) for first driver
  - Schedule delay late and a queue (on bad days) for last driver
  - These must be the same in equilibrium, denote as δ^1

Solving the model

- For first driver:

$$\delta^1 = c_g(t_r) = \beta(t^* - t_r)$$
$$= \beta Q \left\{ \frac{\gamma + p(V_a^1)\left[(\alpha + \gamma)\left(\frac{V_a^1}{V_{K'}} - 1\right)\right]}{(\beta + \gamma)V_a^1} \right\}$$

- Set toll schedule to increases the travel delay and schedule delay for all drivers to δ^1

Optimal toll schedule $\tau(t)$

Early: $t_r \leq t \leq t_{em}^1$

$\delta^1 - \beta(t^* - t) - p(V_a^1)\{(\alpha-\beta)[(V_a^1 / V_{K'}) - 1](t - t_r)\}$

Middle: $t_{em}^1 < t \leq t^*$

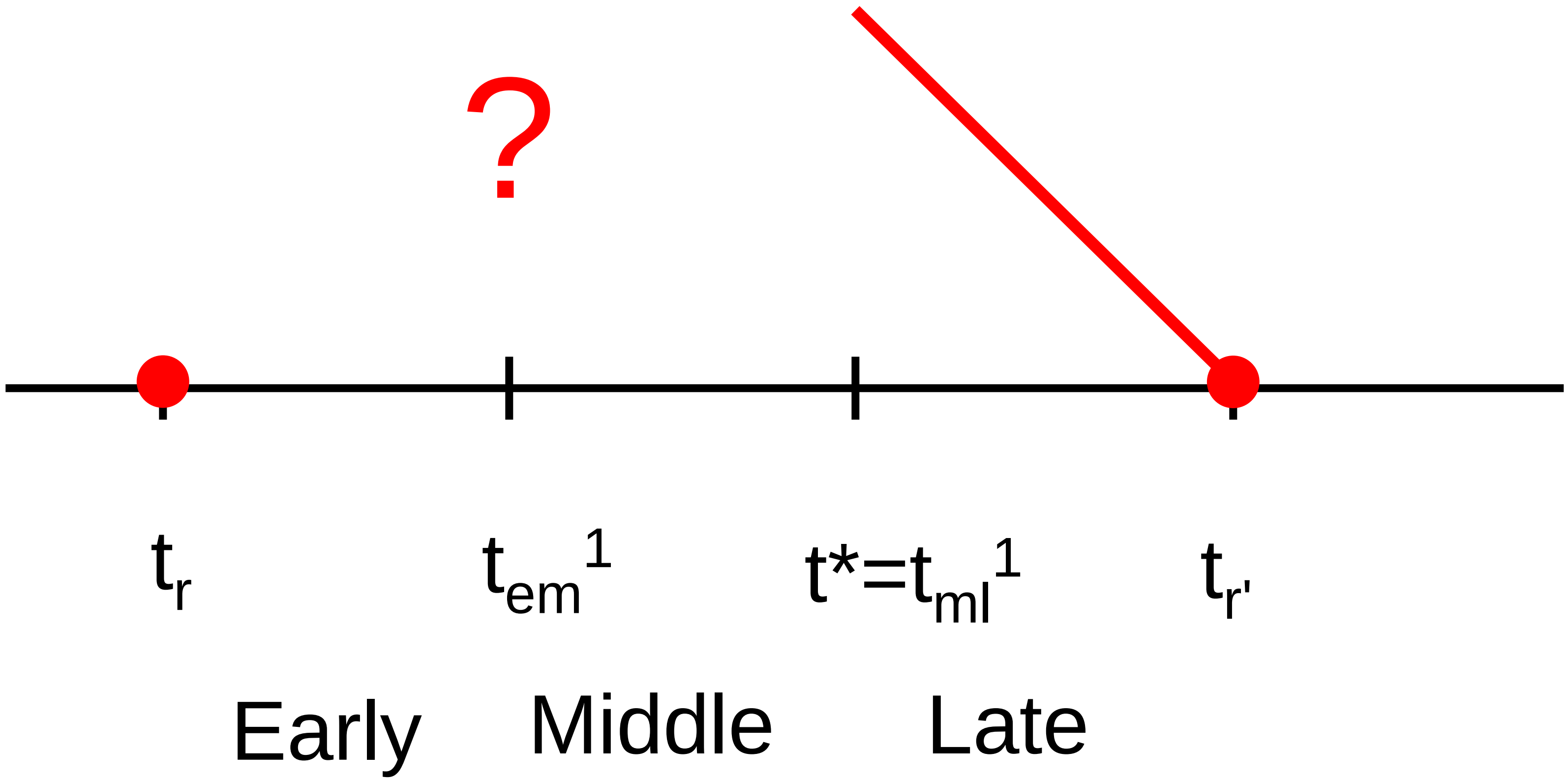
$\delta^1 - [1 - p(V_a^1)]\beta(t^* - t) - p(V_a^1)\gamma(t - t^*)$

$- p(V_a^1)(\alpha+\gamma)[(V_a^1 / V_{K'}) - 1](t - t_r)$

Late: $t^* < t \leq t_{r'}$

$\delta^1 - \gamma(t - t^*) + p(V_a^1)(\alpha+\gamma)[(V_a^1 / V_{K'}) - 1](t - t_r)$

Toll schedule (not to scale)



EXTENSIONS

Possible extensions

- Rather than at the start of the peak, breakdown may occur randomly within the peak
- Number of commuters (Q) is elastic
- Highway congestible (travel time increases with flow) even in a non-breakdown state
- Second best coarse toll

FINAL THOUGHTS

Summary and final thoughts

- Model with endogenous breakdown probability
- Get day-to-day travel time variability without stochastic demand
- Model describes reality where you are generally early or on time but occasionally late
- Applicable if departure time precommitted

Thank you

- Ian Savage: ipsavage@northwestern.edu

- Read the draft paper:

  http://faculty.wcas.northwestern.edu/~ipsavage/440-manuscript.pdf

  Preliminary and incomplete, please do not cite