Uncertainty in Hazardous Materials Transportation

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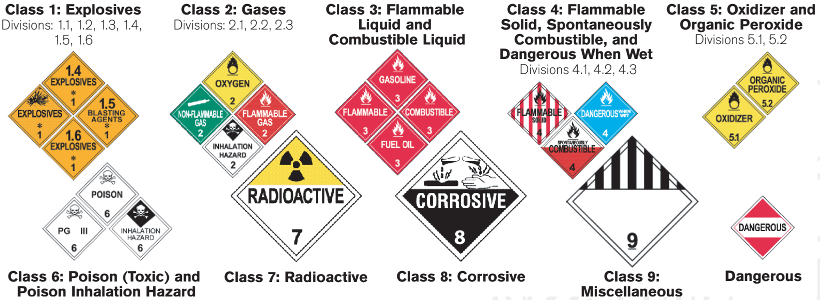
May 7, 2015

Outline

Outline

Hazardous Materials

Hazardous Materials (hazmat), Dangerous Goods



Introduction

Hazmat transportation

- Number of accidents is small compared to the number of shipments
- Consequence is very severe in terms of fatalities, injuries, large-scale evacuation and environmental damage

Introduction

Table: 2014 Hazmat Summary by Transportation Phase [1]

Transportation Phase	Incidents	Hospitalized	Non-Hospitalized	Fatalities	Damages
In Transit	4,190	2	53	5	$63,686,925
In Transit Storage	614	1	1	0	$1,629,889
Loading	3,262	3	20	0	$1,021,289
Unloading	8,149	5	47	1	$3,848,737
Unreported	1	0	0	0	$0

[1] Hazmat Intelligence Portal, US Department of Transportation.

Introduction

Table: Hazmat Shipment Tonnage Shares by Mode in 2007[2]

Mode of Transportation	Percentage of Tons
Truck	53.9%
Pipeline	28.2%
Water	6.7%
Rail	5.8%
Multiple modes	5.0%
Other and unknown modes	0.4%

[2] Research and Innovative Technology Administration and US Census Bureau, *2007 Commodity Flow Survey, Hazardous Materials*

Three Types of Uncertainty

1. Where will be the accident location?
   - Probabilistic nature of traffic accident
2. How large will be the accident consequence?
   - Data uncertainty
3. How do hazmat carriers determine routes?
   - Behavioral uncertainty

- (1), (2): Risk Measures
- (2), (3): Robustness

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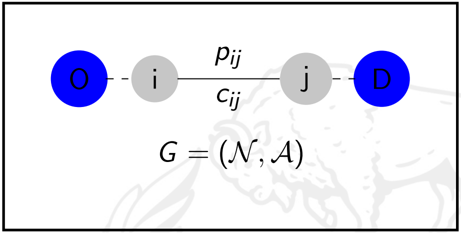
Hazmat Transportation Network

- $G = (\mathcal{N}, \mathcal{A})$ – a road network
- $\mathcal{N}$ is the node set and $\mathcal{A}$ is the arc set.
- p_{ij} – accident probability on arc $(i, j) \in \mathcal{A}$.
- c_{ij} – accident consequence of traveling on arc $(i, j) \in \mathcal{A}$.



Comparison of Risk Measures

Model	Risk Measure	Function
TR	Expected Risk	$\min_{l \in \mathcal{P}} \sum_{(i,j) \in \mathcal{A}^l} p_{ij} c_{ij}$

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IP	Incident Probability	$\min_{l \in \mathcal{P}} \sum_{(i,j) \in \mathcal{A}^l} p_{ij}$
PR	Perceived Risk	$\min_{l \in \mathcal{P}} \sum_{(i,j) \in \mathcal{A}^l} p_{ij} (c_{ij})^q$

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DU	Disutility	$\min_{l \in \mathcal{P}} \sum_{(i,j) \in \mathcal{A}^l} p_{ij}(\exp(k c_{ij} - 1))$
CR	Conditional Probability	$\min_{l \in \mathcal{P}} \left(\sum_{(i,j) \in \mathcal{A}^l} p_{ij} c_{ij} \Big/ \sum_{(i,j) \in \mathcal{A}^l} p_{ij} \right)$

Value-at-Risk (VaR) in Hazmat Problem[3]

- Cutoff Risk β^l_α for path l such that
  - the probability of a shipment experiencing a greater risk than β^l_α is less than confidence level α

$$VaR^l_\alpha = \min\{\beta : \Pr(R^l > \beta) \leq 1 - \alpha\}$$

  - $VaR = 100$ at $\alpha = 99\%$: With probability 99%, risk is less than 100.
- Risk of a path l:

$$R^l = \begin{cases} 0, & \text{w.p.} \quad 1 - \sum_{i=1}^{\overline{m}^l} p_i \\ C_1, & \text{w.p.} \quad p_1 \\ \vdots & \\ C_{\overline{m}^l}, & \text{w.p.} \quad p_{\overline{m}^l} \end{cases}$$

[3] Kang, Y., R. Batta and C. Kwon (2014). "Value-at-Risk Model for Hazardous Material Transportation"

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Risk Preference with VaR

Confidence Level α	0	$\longrightarrow$	1
Risk Attitude	Risk Indifferent	$\longrightarrow$	Risk Averse
Equivalent Model	- -		Min-Max Model $\min_{l \in \mathcal{P}} \max_{(i,j) \in \mathcal{P}^l} C_{(i,j)}$

Sufficiently small α can be as large as 0.999977 in hazmat routing.

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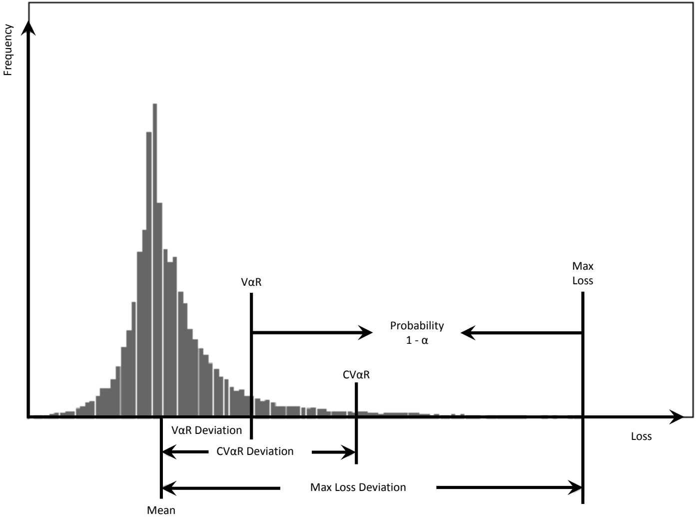
VaR vs Conditional Value-at-Risk (CVaR)



Risk Preference with CVaR

Confidence Level α	0	$\longrightarrow$	1
Risk Attitude	Risk Neutral	$\longrightarrow$	Risk Averse
Equivalent Model	Traditional Risk Model $\min_{l \in \mathcal{P}} \mathbb{E}[R^l]$		Min-Max Model $\min_{l \in \mathcal{P}} \max_{(i,j) \in \mathcal{P}^l} c_{ij}$

CVaR Defined

- For a path $l \in \mathcal{P}$ at the confidence level α, the CVaR is defined as:

$$\text{CVaR}_\alpha^l = \lambda_\alpha^l \text{VaR}_\alpha^l + (1 - \lambda_\alpha^l)\mathbb{E}[R^l : R^l > \text{VaR}_\alpha^l]$$

where $\lambda_\alpha^l = \Big(\Pr[R^l \leq \text{VaR}_\alpha^l] - \alpha\Big) \Big/ (1 - \alpha)$.

- Hard to be considered in an optimization problem format mainly due to conditioning.

Auxiliary Form

- Following Rockafellar and Uraysev (2000), we consider the following function:

$$\Phi_\alpha^l(v) = v + \frac{1}{1-\alpha}\mathbb{E}[R^l - v]^+$$
$$\approx v + \frac{1}{1-\alpha}\sum_{(i,j)\in\mathcal{A}^l} p_{ij}[c_{ij} - v]^+$$

where we denote $[x]^+ = \max(x, 0)$.

- Then, we can show that the CVaR minimization is equilvalent to minimize Φ_α^l by choosing a path $l \in \mathcal{P}$ at the confidence level α. That is,

$$\min_{l\in\mathcal{P}} CVaR_\alpha^l = \min_{l\in\mathcal{P}, v\in\mathbb{R}^+} \Phi_\alpha^l(v)$$

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A Computational Method for CVaR Minimization

- $v^* \in \{0\} \cup \{c_{ij} : (i,j) \in \mathcal{A}\}$
- A shortest-path algorithm (like Dijkstra's) for solving the sub-problem.
- $|\mathcal{A}| + 1$ number of shortest-path problems.

- C. Kwon (2011), "Conditional Value-at-Risk Model for Hazardous Materials Transportation", in *Proceedings of the 2011 Winter Simulation Conference*, S. Jain, R. R. Creasey, J. Himmelspach, K. P. White, and M. Fu, eds. pp. 1708-1714
- Toumazis, I., C. Kwon, and R. Batta (2013), "Value-at-Risk and Conditional Value-at-Risk Minimization for Hazardous Materials Routing", in *Handbook of OR/MS Models in Hazardous Materials Transportation* (Eds.:R. Batta and C. Kwon), Springer
- Toumazis, I. and C. Kwon (2013), "Routing Hazardous Materials on Time-Dependent Networks using Conditional Value-at-Risk", *Transportation Research Part C: Emerging Technologies*, 37, 7392.

Outline

A Challenge for Hazmat Routing

- Accident Probability: data is usually unavailable.
- Accident Consequence: any measure is hard to quantify and subject to uncertainty.
- Need a robust method for data uncertainty.

Shortest Path Problem

For a graph $G(\mathcal{N}, \mathcal{A})$, to find a path with the least path cost.

$$\min_{x \in \Omega} \sum_{(i,j) \in \mathcal{A}} c_{ij} x_{ij}$$

where

$$\Omega \equiv \Bigg\{ x : \sum_{(i,j) \in \mathcal{A}} x_{ij} - \sum_{(j,i) \in \mathcal{A}} x_{ji} = b_i \quad \forall i \in \mathcal{N}, \\ \text{and } x_{ij} \in \{0, 1\} \quad \forall (i,j) \in \mathcal{A} \Bigg\}$$

Dijkstra's Algorithm $O(|\mathcal{N}|^2)$

Robust Shortest Path Problem

The cost coefficient c may be subject to some uncertainty.

$$\min_{x \in \Omega} \sum_{(i,j) \in \mathcal{A}} \tilde{c}_{ij} x_{ij}$$

The uncertain $\tilde{c}$ belongs to an uncertain set C.

$$\min_{x \in \Omega} \max_{\tilde{c} \in C} \sum_{(i,j) \in \mathcal{A}} \tilde{c}_{ij} x_{ij}$$

The robust shortest path problem is to find a path that minimizes the worst-case path cost.

Two Mulplicative Cost Coefficients[4]

Nominal Problem

$$\min_{x \in \Omega} \sum_{(i,j) \in \mathcal{A}} p_{ij} c_{ij} x_{ij}$$

Uncertain Problem

$$\min_{x \in \Omega} \sum_{(i,j) \in \mathcal{A}} \tilde{p}_{ij} \tilde{c}_{ij} x_{ij}$$

Robust Problem

$$\min_{x \in \Omega} \max_{\tilde{p}, \tilde{c}} \sum_{(i,j) \in \mathcal{A}} \tilde{p}_{ij} \tilde{c}_{ij} x_{ij}$$

[4] Kwon, C., T. Lee, P. G. Berglund (2013), "Robust Shortest Path Problems with Two Uncertain Multiplicative Cost Coefficients", *Naval Research Logistics*, 60(5), 375394

Robust Shortest Path Problems with Two Uncertain Multiplicative Cost Coefficients

$$\min_{x\in\Omega} \max_{u\in U, v\in V} \sum_{(i,j)\in\mathcal{A}} (p_{ij} + q_{ij}u_{ij})(c_{ij} + d_{ij}v_{ij})x_{ij}$$

where

$$U = \left\{ u : 0 \le u_{ij} \le 1 \quad \forall (i,j), \quad \sum_{(i,j)} u_{ij} \le \Gamma_u \right\}$$

$$V = \left\{ v : 0 \le v_{ij} \le 1 \quad \forall (i,j), \quad \sum_{(i,j)} v_{ij} \le \Gamma_v \right\}$$

and Γ_u and Γ_v are positive integers.

The objective function can be written as follows:

$$\min_{x\in\Omega} \left[p_{ij}c_{ij}x_{ij} + \max_{u\in U, v\in V} (q_{ij}c_{ij}x_{ij}u_{ij} + p_{ij}d_{ij}x_{ij}v_{ij} + q_{ij}d_{ij}x_{ij}u_{ij}v_{ij}) \right]$$

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Mixed Integer Linear Program

$$\min p_{ij} c_{ij} x_{ij} + \Gamma_u \theta_u + \Gamma_v \theta_v + \sum_{(i,j)} (\rho_{ij} + \mu_{ij})$$

subject to

$$
\begin{aligned}
x &\in \Omega \\
\rho_{ij} - \eta_{ij} + \theta_u &\geq q_{ij} c_{ij} x_{ij} \\
\mu_{ij} - \pi_{ij} + \theta_v &\geq p_{ij} d_{ij} x_{ij} \\
\eta_{ij} + \pi_{ij} &\geq q_{ij} d_{ij} x_{ij} \\
\rho_{ij}, \mu_{ij}, \eta_{ij}, \pi_{ij}, \theta_u, \theta_v &\geq 0
\end{aligned}
$$

Solution of the Dual Problem

A solution to the dual problem for any given x is:

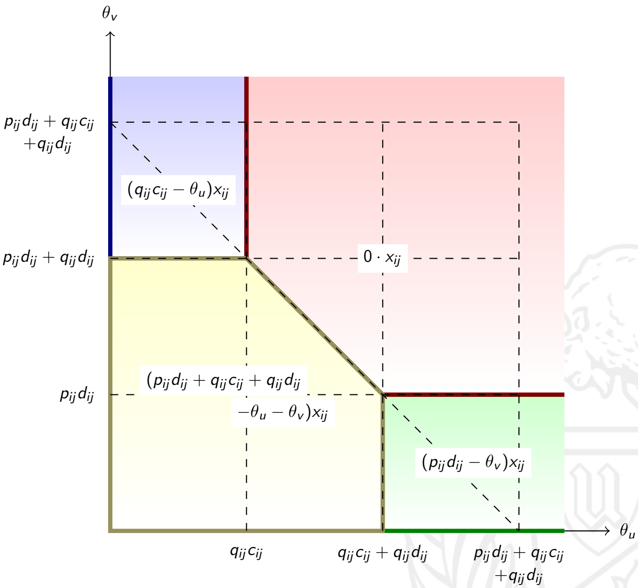
$$\begin{aligned}\rho_{ij} &= \max(q_{ij}c_{ij}x_{ij} - \theta_u + \eta_{ij}, 0) \\ &= \max(q_{ij}c_{ij}x_{ij} + q_{ij}d_{ij}x_{ij} - \min(q_{ij}d_{ij}x_{ij}, \max(\theta_v - p_{ij}d_{ij}x_{ij}, 0)) \\ &\qquad - \theta_u, 0) \\ \mu_{ij} &= \max(p_{ij}d_{ij}x_{ij} - \theta_v + \pi_{ij}, 0) \\ &= \max(p_{ij}d_{ij}x_{ij} + \min(q_{ij}d_{ij}x_{ij}, \max(\theta_v - p_{ij}d_{ij}x_{ij}, 0)) - \theta_v, 0)\end{aligned}$$

Lemma

The sum $\rho_{ij} + \mu_{ij}$ can be expressed as follows:

$$\rho_{ij} + \mu_{ij} = \begin{cases} 0 \cdot x_{ij} \\ \quad \text{if } \theta_u \geq q_{ij}c_{ij},\ \theta_v \geq p_{ij}d_{ij} + q_{ij}d_{ij} \\ \quad \text{or } p_{ij}d_{ij} \leq \theta_v \leq p_{ij}d_{ij} + q_{ij}d_{ij},\ \theta_u + \theta_v \geq p_{ij}d_{ij} + q_{ij}c_{ij} + q_{ij}d_{ij} \\ \\ (q_{ij}c_{ij} - \theta_u)x_{ij} & \text{if } \theta_u \leq q_{ij}c_{ij}, \theta_v \geq p_{ij}d_{ij} + q_{ij}d_{ij} \\ \\ (p_{ij}d_{ij} + q_{ij}c_{ij} + q_{ij}d_{ij} - \theta_u - \theta_v)x_{ij} \\ \quad \text{if } p_{ij}d_{ij} \leq \theta_v \leq p_{ij}d_{ij} + q_{ij}d_{ij},\ \theta_u + \theta_v \leq p_{ij}d_{ij} + q_{ij}c_{ij} + q_{ij}d_{ij} \\ \quad \text{or } \theta_u \leq q_{ij}c_{ij} + q_{ij}d_{ij}, \theta_v \leq p_{ij}d_{ij} \\ \\ (p_{ij}d_{ij} - \theta_v)x_{ij} & \text{if } \theta_u \geq q_{ij}c_{ij} + q_{ij}d_{ij}, \theta_v \leq p_{ij}d_{ij} \end{cases}$$

for each $(i, j) \in \mathcal{A}$ and all $\theta_u \geq 0$ and $\theta_v \geq 0$.


θ_v
$p_{ij}d_{ij} + q_{ij}c_{ij}$
$+q_{ij}d_{ij}$
$(q_{ij}c_{ij} - \theta_u)x_{ij}$
$p_{ij}d_{ij} + q_{ij}d_{ij}$
$0 \cdot x_{ij}$
$(p_{ij}d_{ij} + q_{ij}c_{ij} + q_{ij}d_{ij}$
$p_{ij}d_{ij}$
$-\theta_u - \theta_v)x_{ij}$
$(p_{ij}d_{ij} - \theta_v)x_{ij}$
θ_u
$q_{ij}c_{ij}$
$q_{ij}c_{ij} + q_{ij}d_{ij}$
$p_{ij}d_{ij} + q_{ij}c_{ij}$
$+q_{ij}d_{ij}$

- Let $\{a_k\}$ be the ordered sequence of $q_{ij}c_{ij} + q_{ij}d_{ij}$ and $q_{ij}c_{ij}$ for all $(i,j) \in \mathcal{A}$ and 0.
- Let $\{b_l\}$ be the ordered sequence of $p_{ij}d_{ij}$ and $p_{ij}d_{ij} + q_{ij}d_{ij}$ for all $(i,j) \in \mathcal{A}$ and 0.
- Let $\{f_m\}$ be the ordered sequence of $p_{ij}d_{ij} + q_{ij}c_{ij} + q_{ij}d_{ij}$ for all $(i,j) \in \mathcal{A}$ and 0.

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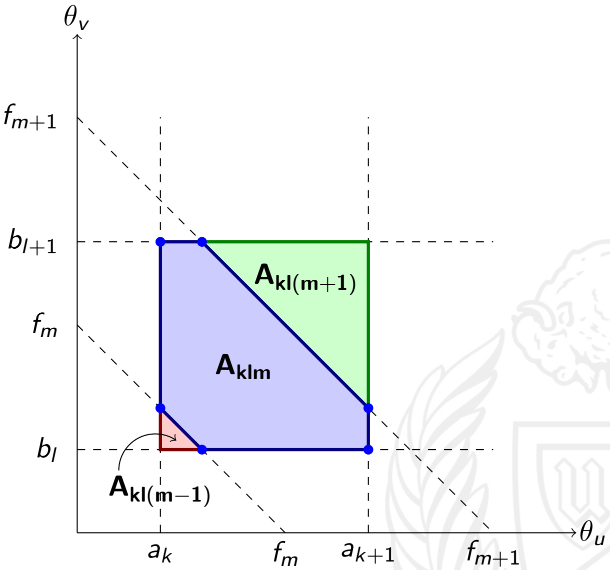
We consider the following problem, defined over a sub-area of the entire (θ_u, θ_v)-space:

$$Z_{klm} = \min_{x, \theta_u, \theta_v} \Gamma_u \theta_u + \Gamma_v \theta_v + \sum_{(i,j)} (p_{ij} c_{ij} x_{ij} + \rho_{ij} + \mu_{ij})$$

subject to

$$\begin{aligned} x &\in \Omega \\ a_k \leq \theta_u &\leq a_{k+1} \\ b_l \leq \theta_v &\leq b_{l+1} \\ f_m \leq \theta_u + \theta_v & \qquad \text{if } f_m \in [a_k + b_l, a_{k+1} + b_{l+1}] \\ f_{m+1} \geq \theta_u + \theta_v & \qquad \text{if } f_{m+1} \in [a_k + b_l, a_{k+1} + b_{l+1}] \end{aligned}$$


θ_v
f_{m+1}
b_{l+1}
f_m
b_l
A_{kl(m+1)}
A_{klm}
A_{kl(m−1)}
a_k
f_m
a_{k+1}
f_{m+1}
θ_u

Robust Problem

Theorem

Let us define the following problem with an arbitrary constraint set Θ*:*

$$Z(\Theta) = \min_{x \in \Omega, (\theta_u, \theta_v) \in \Theta} \Gamma_u \theta_u + \Gamma_v \theta_v + \sum_{(i,j)} (p_{ij} c_{ij} x_{ij} + \rho_{ij} + \mu_{ij}) \quad (1)$$

Then the robust shortest path problem is equivalent to the following problem:

$$Z^* = \min_{k,l,m} Z(\Theta_{klm}) \quad (2)$$

Some reduction in the search space is possible! [5]

[5] Kwon, C., T. Lee, P. G. Berglund (2013), "Robust Shortest Path Problems with Two Uncertain Multiplicative Cost Coefficients", *Naval Research Logistics*, 60(5), 375394

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Theorem

Let us define the following problem with an arbitrary constraint set Θ:

$$Z(\Theta) = \min_{x \in \Omega, (\theta_u, \theta_v) \in \Theta} \Gamma_u \theta_u + \Gamma_v \theta_v + \sum_{(i,j)} (p_{ij} c_{ij} x_{ij} + \rho_{ij} + \mu_{ij}) \quad (1)$$

Then the robust shortest path problem is equivalent to the following problem:

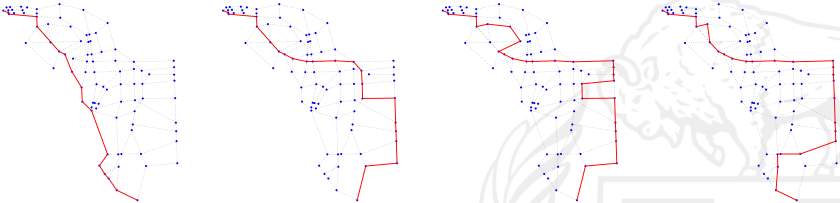
$$Z^* = \min_{k,l,m} Z(\Theta_{klm}) \quad (2)$$

Some reduction in the search space is possible! [5]

[5] Kwon, C., T. Lee, P. G. Berglund (2013), "Robust Shortest Path Problems with Two Uncertain Multiplicative Cost Coefficients", *Naval Research Logistics*, 60(5), 375394

Worst-Case Conditional Value-at-Risk (WCVaR) [6]

We can also consider the worst-case of the CVaR risk measure. Computing the best WCVaR route requires solving a series of robust shortest-path problems.



$\alpha = 0$ $\alpha = 0.999965$ $\alpha = 0.999973$ $\alpha = 0.999990$

[6] Toumazis, I. and C. Kwon, "Worst-case Conditional Value-at-Risk Minimization for Hazardous Materials Transportation", submitted to *Transportation Science*, in Revision

Outline

Hazmat Network Design

- To design a safe road network, considering drivers' reaction to the design change.

$$\min_{y} \text{Risk}(x(y))$$

  where $x(y)$ describes the drivers' reaction to the design variable y.
- y is a binary variable to close a certain link or not.
- Uncertainty in the risk measure can be considered. [7]
- Most papers assume drivers take the shortest path, i.e., $x(y)$ is a solution to the shortest-path problem.

---

[7] Sun, L., M. Karwan, and C. Kwon, "Robust Hazmat Network Design Problems Considering Risk Uncertainty", submitted to *Transportation Science*, in Revision

Behavioral Uncertainty

- Zhu and Levinson (2010): Most commuters do not choose the shortest path
- Nakayama et al. (2001): drivers are not fully rational
- How do hazmat drivers choose routes? Travel time? Highway vs local roads? Number of turns?
- We want some robustness in hazmat network design against behavioral uncertainty of drivers.

Example: two groups of people

- Betty Rogerson (BR): Doesn't care about the shortest path, as long as her path is within 5 minute difference.
- Peter Edison (PE): Cares about the shortest path, but based on his own perception of link travel costs.

Bounded Rationality in Transportation

- Simon (1955): "A Behavioral Model of Rational Choice"
- Drivers choose a route if its length is no longer than the shortest-path length + a certain threshold
- Mahmassani and Chang (1987): "A boundedly rational user equilibrium (BRUE) is achieved in a transportation system when all users are satisfied with their current travel choices."
- Han et al. (2014): dynamic BRUE
- Lou et al. (2010): robust congestion pricing with BR
- This presentation: perception error model to generalize BR in the context of hazmat transportation (on-going research)

Bounded Rationality

Definition (Additive Bounded Rationality)

A path is called a boundedly rational shortest path within an additive indifference band, if the path can be represented by a vector $x \in X$ such that

$$\text{(A-BR)} \qquad \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} \le c^0 + E \tag{3}$$

where E is a positive constant for the additive indifference band.

Definition (Multiplicative Bounded Rationality)

A path is called a boundedly rational shortest path within a multiplicative indifference band, if the path can be represented by a vector $x \in X$ such that

$$\text{(M-BR)} \qquad \sum_{(i,j)\in\mathcal{A}} c_{ij}x_{ij} \le (1+\kappa)c^0 \tag{4}$$

where $\kappa \in (0,1)$ is a constant for the multiplicative indifference band.

Perception Error

- The Perception Error (PE) model:

$$\text{(PE)} \qquad \min_{x \in X} \sum_{(i,j) \in \mathcal{A}} (c_{ij} - \varepsilon_{ij}) x_{ij} \tag{5}$$

  for some constant cost vector $\varepsilon \in \mathcal{E}$.

- ε: a vector of perception error of link cost
- $\mathcal{E}$: set of uncertain perception error

Equivalence of BR and PE

$$\text{(PE)} \qquad \min_{x \in X} \sum_{(i,j)\in\mathcal{A}} (c_{ij} - \varepsilon_{ij}) x_{ij} \tag{6}$$

$$\mathcal{E}_A = \left\{ \varepsilon : \sum_{(i,j)\in\mathcal{A}} \varepsilon_{ij} \leq E, \quad \varepsilon_{ij} \geq 0 \ \forall (i,j) \in \mathcal{A} \right\} \tag{7}$$

$$\mathcal{E}_M = \left\{ \varepsilon : \sum_{(i,j)\in\mathcal{A}} \varepsilon_{ij} \leq \kappa c^0, \quad \varepsilon_{ij} \geq 0 \ \forall (i,j) \in \mathcal{A} \right\} \tag{8}$$

Theorem

- $\text{PE} + \mathcal{E}_A \iff$ A-BR
- $\text{PE} + \mathcal{E}_M \iff$ M-BR

Sub-path Multiplicative Bounded Rationality (SM-BR)

Definition

A path is called a *subpath multiplicative bounded-rationality* shortest-path, or *SM-BR* path with the multiplicative indifference band κ, if any subpath of the path is an M-BR path with the same multiplicative indifference band κ between the corresponding origin and destination nodes.

$$\mathcal{E}_L = \left\{ \varepsilon : 0 \leq \varepsilon_{ij} \leq \frac{\kappa}{1+\kappa} c_{ij} \ \forall (i,j) \in \mathcal{A} \right\} \tag{9}$$

Theorem

$\text{PE} + \mathcal{E}_L \iff \text{SM-BR}$

Some Other Examples of the Perception Error Set $\mathcal{E}$

$$\mathcal{E}_H = \left\{ \varepsilon : \sum_{(i,j)\in\mathcal{A}} \varepsilon_{ij} \le E, \sum_{(i,j)\in\mathcal{A}} \varepsilon_{ij} \le (1+\kappa)c^0 \quad \forall (i,j) \in \mathcal{A} \right\} \tag{10}$$

$$\mathcal{E}_B = \left\{ \varepsilon : l_{ij} \le \varepsilon_{ij} \le u_{ij} \le c_{ij} \quad \forall (i,j) \in \mathcal{A} \right\} \tag{11}$$

Ellipsoidal Set

$$\mathcal{E}_E = \left\{ \varepsilon : ||Q^{-1/2}\varepsilon||_2 \leq \xi \right\}$$

Theorem

Let $\bar{x}$ be an optimal solution to (6) for some $\varepsilon \in \mathcal{E}_E$. Then,

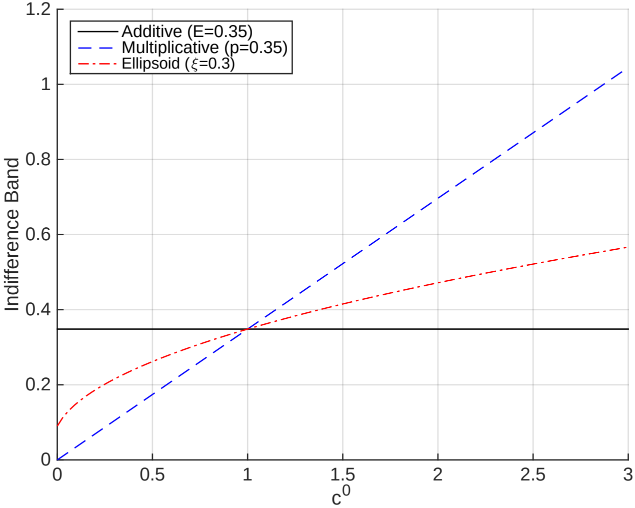
$$\sum_{(i,j)\in\mathcal{A}} c_{ij}\bar{x}_{ij} \leq c^0 + \xi\sqrt{\bar{x}^T Q \bar{x}} \tag{12}$$

Furthermore, the following bound holds

$$\sum_{(i,j)\in\mathcal{A}} c_{ij}\bar{x}_{ij} \leq c^0 + \frac{\xi^2}{2} + \xi\sqrt{c^0 + \frac{\xi^2}{4}} \tag{13}$$

in a special case when $Q = \text{diag}(\ldots, c_{ij}, \ldots)$.

Ellipsoidal Set



Generalized Bounded Rationality

- BR is a special case of PE.
- With PE, modelers have flexibility with link-specific preferences/perception of drivers.
- Link-based modeling (PE) is usually preferred to path-based modeling (BR).

Definition

A network user possesses generalized bounded rationality, if the user's route-choice decision-making can be justified by the perception error model for some closed and bounded set $\mathcal{E}$.

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Robust Network Design with PE

$$\min_{y} \max_{x,\varepsilon} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} r_{ij}^s x_{ij}^s \tag{14}$$

subject to

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in \mathcal{A} \tag{15}$$

$$\varepsilon^s \in \mathcal{E}^s \quad \forall s \in \mathcal{S} \tag{16}$$

$$x^s = \arg\min_{x} \sum_{(i,j)\in\mathcal{A}} (c_{ij} - \varepsilon_{ij}^s) x_{ij}^s \tag{17}$$

$$\text{subject to} \quad -\sum_{(i,j)\in\mathcal{A}} x_{ij}^s + \sum_{(j,i)\in\mathcal{A}} x_{ji}^s = -b_i^s \quad \forall i \in \mathcal{N} \tag{18}$$

$$x_{ij}^s \le y_{ij} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{19}$$

$$x_{ij}^s \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{20}$$

Cutting Plane Algorithm

Step 1. Solve the master network design problem to obtain x^k and y^k.

$$
\begin{aligned}
\min_y \ & \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} r_{ij}^s x_{ij}^s \\
\text{s.t. } & y_{ij} \in \{0,1\} \quad \forall (i,j)\in\mathcal{A} \\
& \varepsilon^s \in \mathcal{E}^s \quad \forall s\in\mathcal{S} \\
& \min_x \sum_{(i,j)\in\mathcal{A}} (c_{ij} - \varepsilon_{ij}^s) x_{ij}^s \\
& \text{s.t. } \quad - \sum_{(i,j)\in\mathcal{A}} x_{ij}^s + \sum_{(j,i)\in\mathcal{A}} x_{ji}^s = -b_i^s \quad \forall i\in\mathcal{N} \\
& \qquad x_{ij}^s \le y_{ij} \quad \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} \\
& \qquad x_{ij}^s \in \{0,1\} \quad \forall (i,j)\in\mathcal{A}, s\in\mathcal{S}
\end{aligned}
$$

Cutting Plane Algorithm

Step 2. Given y^k, solve the worst-risk route-choice problem and obtain $\hat{x}^k$:

$$\max_{x,\varepsilon} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} r_{ij}^s x_{ij}^s$$

$$\text{s.t. } \varepsilon^s \in \mathcal{E}^s \quad \forall s \in \mathcal{S}$$

$$\min_x \sum_{(i,j)\in\mathcal{A}} (c_{ij} - \varepsilon_{ij}^s) x_{ij}^s$$

$$\text{s.t.} \quad -\sum_{(i,j)\in\mathcal{A}} x_{ij}^s + \sum_{(j,i)\in\mathcal{A}} x_{ji}^s = -b_i^s \quad \forall i \in \mathcal{N}$$

$$x_{ij}^s \le y_{ij}^k \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

$$x_{ij}^s \in \{0,1\} \quad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

Cutting Plane Algorithm

Step 3. If $\hat{x}^k$ is identical to x^k, stop. Otherwise add cuts to the master network design problem and go to Step 1.

$$\sum_{(i,j)\in p} x_{ij}^s \leq |p| - 1 + z_p,$$

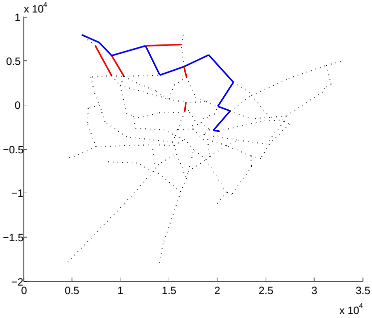
$$z_p \leq x_{ij}^s \quad \forall (i,j) \in p,$$

$$\sum_{(i,j)\in p'} y_{ij} \leq |p'| - z_p,$$

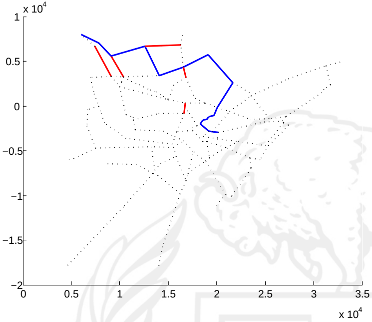
$$z_p \in \{0, 1\}$$

These cuts make the path $\hat{x}^k$ unavailable.

Solution without considering uncertain behavior

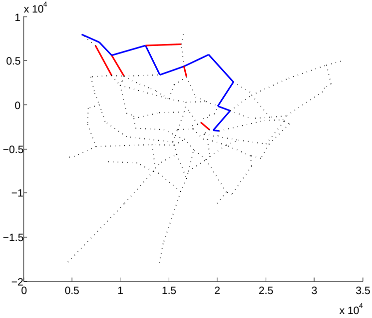


Nominal design

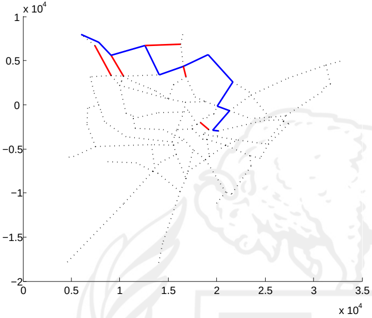


Worst-case

Robust solution considering uncertain behavior



Robust design



Worst-case

Question???

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